PLANE CAUCHY–POISSON PROBLEM FOR A POND WITH A SMOOTHLY CHANGING BOTTOM. EXAMPLES OF NUMERICAL COMPUTATIONS

A. A. Korobkin and I. V. Sturova

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1. Unsteady motion in a gravity force field is considered for a layer of a homogeneous imcompressible ideal fluid of variable depth caused by an initial perturbation of the free boundary. The pattern of the motion is described completely by the velocity potential $\varphi(x, y, t)$ which satisfies the following relationships in a linear formulation

$$\Delta\varphi = 0 \text{ in } \Omega,\ \varphi_{tt} + \varphi_y = 0\ (y = 0,\ x \in R^1),$$
$$\varphi_y + h_x\varphi_x = 0\ (y = -h(x),\ x \in R^1),\ \varphi = 0,\ \varphi_t = -f(x)\ (y = 0,\ t = 0). \tag{1.1}$$

The y axis in a Cartesian (x, y) coordinate system is directed opposite to the free-fall direction. The relationships (1.1) are written in dimensionless variables, where the length and velocity scales are selected so that the Froude number of the problem and the fluid depth at x = 0 equal one. At the initial time (t = 0) the free surface of the fluid is deflected from its equilibrium position. The equations $y = f(x)$, $y = -h(x)$, $x \in R^1$ describe the initial position of the free boundary and the shape of the pond bottom, respectively, $[h(x) > 0]$.

Under the assumption of a smooth change in the pond depth, i.e., $\varepsilon = \max_{x \in R^1} |h_x| \ll 1$, the fundamental solution of the formulated problem is constructed and investigated preliminarily in [1]. The purpose of this paper is to illustrate the solution obtained by numerical computations and to investigate the nature of the influence of different bottom roughnesses on the behavior of the free surface. An analysis of the vertical displacements of the free surface $\eta(x, t)$ that are determined from the relationship $\eta = -\varphi_t(x, 0, t)$ is of main interest. For simplicity, evenness of the function f(x) is assumed.

In the case of a level bottom ($h \equiv 1$), the formula describing the evolution of the free boundary

$$\eta_0(x, t) = \frac{1}{\pi}\int_0^\infty F(\nu)\cos\nu x\cos\Omega(\nu)t\,d\nu \tag{1.2}$$

$$\left(\Omega(\nu) = (\nu\,\mathrm{th}\,\nu)^{1/2},\ F(\nu) = 2\int_0^\infty f(x)\cos\nu x\,dx\right)$$

is well known. For waves being propagated to the right $[t \to \infty,\ x/t = O(1),\ x > 0]$ for large times the following asymptotic is valid:

$$\eta_0(x, t) = F(\alpha)(2\pi t|\Omega''(\alpha)|)^{-1/2}\sin[\Omega(\alpha)t + \alpha x + \pi/4] + O(t^{-1}), \tag{1.3}$$

which is obtained by using the method of stationary phase. Here $\alpha = \alpha(x/t)$ is the solution of the equation $\Omega'(\alpha) = -x/t$. The asymptotic (1.3) is not uniform; it describes the surface wave pattern far from the bow wave front [1], i.e., does not even yield information about the free surface shape for $x \geq t$, $t \gg 1$ for $0 < x < t$.

If the pond depth varies slowly ($\varepsilon \ll 1$), then to $O(\varepsilon^{3/2})$ accuracy we have [1]

$$\eta(x, t) = 2\Omega(\alpha)C_0(\alpha, \lambda)\,\mathrm{ch}[qh(x)]\sin\theta(\alpha, \lambda), \tag{1.4}$$

where the functions q, θ, C_0 depend on the parameters α, λ ($\alpha < 0$, $\lambda > 0$) and are determined as the solutions of the system of six first-order ordinary differential equations

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$$\frac{dq}{d\lambda}=\frac{q^2h'(x)}{\operatorname{ch}^2 qh(x)},\ \frac{d\theta}{d\lambda}=2\Omega^2(\alpha)-qS(qh),\ \frac{dx}{d\lambda}=-S(qh),\ \frac{dC_0}{d\lambda}=$$
$$=-E(\lambda,\alpha)C_0,\ \frac{dq_\alpha}{d\lambda}=\frac{q}{\operatorname{ch}^2 qh}[2q_\alpha h'+qh''x_\alpha-2qh'\operatorname{th}qh\,(q_\alpha h+qh'x_\alpha)],$$
$$\frac{dx_\alpha}{d\lambda}=-S'(qh)(hq_\alpha+qh'x_\alpha),\ S(\xi)=\operatorname{th}\xi+\frac{\xi}{\operatorname{ch}^2\xi},\ S_1=\frac{S(qh)}{4\Omega^2(\alpha)}+h\operatorname{th}qh, \tag{1.5}$$
$$E(\lambda,\alpha)=q_x[S_1S(qh)-h]-qh'+\frac{q^2h'}{\operatorname{ch}^2 qh}S_1,\ q_x=q_\lambda\lambda_x+q_\alpha\alpha_x,$$
$$\lambda_x=\lambda\Omega'(\alpha)/D,\ \alpha_x=-\Omega(\alpha)/D,\ D=\lambda\Omega'(\alpha)x_\lambda-x_\alpha\Omega(\alpha),\ t=2\lambda\Omega(\alpha).$$

Let us note that the corresponding system was written down in [1] with respect to the "slow" variables εx, εt. The amplitude function C_0 becomes infinitely large as $\lambda\to 0$; consequently, the numerical solution of the system (1.5) is started conveniently for a certain small value of $\lambda=\lambda_0>0$ by using the solution (1.5) for a level bottom as initial conditions:

$$q=\alpha,\ \theta=\lambda_0[2\Omega^2(\alpha)-\alpha S(\alpha)]+\pi/4,\ x=-\lambda_0 S(\alpha),$$
$$C_0=\frac{F(\alpha)}{4\sqrt{\pi\lambda_0|\Omega''(\alpha)|\Omega^3(\alpha)}\operatorname{ch}\alpha},\ q_\alpha=1,\ x_\alpha=-\lambda_0 S'(\alpha). \tag{1.6}$$

Substitution of Eq. (1.6) into Eq. (1.4) results in Eq. (1.3). Numerical integration of the system (1.5) with the initial conditions (1.6) is performed for different α ($\alpha<0$). The function $x_f=x(t)$ obtained as $\alpha\to-0$ determines the boundary of the wave perturbation front being propagated to the right.

2. The plane Cauchy–Poisson problem for a pond with a rough bottom is investigated in [2] under the assumption of small roughnesses. The solution is obtained in the form of the sum

$$\eta=\eta_0(x,t)+\eta_1(x,t),$$

where η_0 is the solution (1.2) for a level bottom and

$$\eta_1=\frac{2}{\pi^2}\int_0^\infty\frac{kF(k)\,dk}{\operatorname{ch}k}\int_0^\infty\frac{pZ(k,p)[\cos\Omega(k)t-\cos\Omega(p)t]}{\operatorname{ch}p\,[\Omega^2(p)-\Omega^2(k)]}dp$$
$$(Z(k,p)=\operatorname{Re}[ipe^{ipx}K(p,k)], \tag{2.1}$$
$$K(p,k)=\int_{-\infty}^{\infty}h_1(x)e^{-ipx}\sin kx\,dx,\ h_1(x)=1-h(x)).$$

The necessary condition for applicability of this approximation is integrability of the function $|h_1(x)|$, which is known to be satisfied for localized bottom roughness.

Comparison of the numerical computations obtained in the small-roughness approximation and in the smoothly-varying bottom approximation is of interest. The case is examined when the shape of the initial perturbation of the free boundary is described by the function

$$f(x)=ae^{-dx^2}, \tag{2.2}$$

and the bottom relief by the function

$$h_1(x)=\begin{cases}b\cos(\pi(x-x_0)/2x_1), & |x-x_0|<x_1;\\ 0, & |x-x_0|>x_1.\end{cases}$$

Let us note that in the small roughness approximation the ratio $\Delta\eta=\eta_1/b$ is independent of b. The dependences $\Delta\eta(x,t)$ are represented in Fig. 1 for different b, here $d=3$, $x_0=7$, $x_1=3$. The solution (2.1) is shown by the solid line, the curves 1-4 correspond to the smooth roughness approximation for $b=-0.2, -0.1, 0.1, 0.2$. This approximation does not describe the reflected waves, meaning $\Delta\eta\equiv 0$ to the left of the roughness (Fig. 1a, $x=3$). The small roughness approximation describes reflected waves but their amplitude is negligible in the case under consideration. Both approximations are in satisfactory agreement above the roughness apex for $x=7$ (Fig. 1b) and behind it for $x=11$ (Fig. 1c). As should have been expected, the agreement is improved as $|b|$ diminishes.

3. The asymptotic (1.4) is not valid near the bow wave front [1], namely, $C_0(\alpha,\lambda)\to\infty$ as $\alpha\to-0$. To refine the shape of the free boundary in the neighborhood of the front, a method proposed by Whitham is used in [1]. However, the composite asymptotic expansion

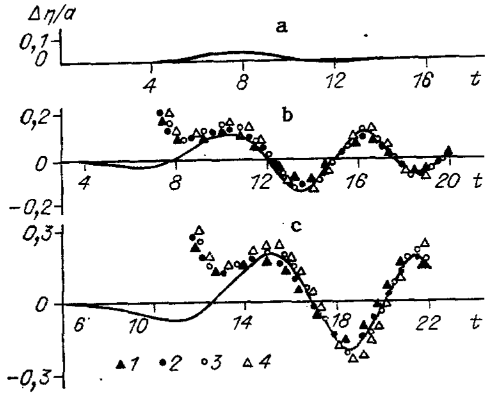

Fig. 1

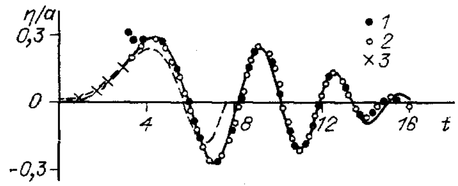

Fig. 2

obtained here (for $\varepsilon \to 0$) at moderate times differs noticeably from the exact solution. The asymptotic formulas for $\eta(x, t)$ as $t \to \infty$, $x/t = O(1)$, are uniformly suitable in the spatial variable x and convenient for numerical computations and can be obtained by using a generalized stationary phase method [3]. In contrast to the usual stationary phase method, it provides the possibility for merging the stationary phase points. Firstly, it is necessary to construct a uniform asymptotic $\eta_0(x, t)$ in the variable x that is defined by Eq. (1.2) as $t \to \infty$. Let us rewrite Eq. (1.2) in the form

$$\eta_0(x,t)=\frac{1}{4\pi}\int_{-\infty}^{\infty} F(\nu)\,e^{it(\Omega_1(\nu)-\nu\sigma)}\,d\nu+\frac{1}{4\pi}\int_{-\infty}^{\infty} F(\nu)\,e^{it(\Omega_1(\nu)+\nu\sigma)}\,d\nu,$$

where $\Omega_1(\nu) = \sqrt{\nu\tanh\nu}\,\operatorname{sgn}\nu$, $\sigma = x/t$. For waves being propagated to the right ($x > 0$) the second integral is of the order of $O(t^{-N})$ as $t \to \infty$ for any $N > 0$. An analogous estimate is valid for the first integral for $\sigma > 1$. If $0 < \sigma \le 1$, then the first integral has two stationary phase points ν_1, ν_2, such that $\Omega_1'(\nu_{1,2}) = \sigma$. The function $\Omega_1(\nu)$ is odd; consequently, $\nu_2 = -\nu_1$, $\nu_1(\sigma) > 0$. Let us introduce a new variable of integration ζ, instead of ν, such that

$$\nu\sigma-\Omega_1(\nu)=(1/3)\zeta^3-B(\sigma)\zeta. \tag{3.1}$$

For the substitution to be nondegenerate it is necessary that $d\zeta(\nu, \sigma)/d\nu \ne 0, \pm\infty$ for $\nu \in R^1$, $0 < \sigma \le 1$. But

$$[\zeta^2-B(\sigma)]d\zeta/d\nu=\sigma-\Omega_1'(\nu),$$

consequently $B(\sigma) = \zeta^2(\nu_1(\sigma), \sigma)$. Substituting the last equality into Eq. (3.1) we find

$$B(\sigma)=(3/2)^{2/3}[\Omega(\nu_1(\sigma))-\sigma\nu_1(\sigma)]^{2/3}.$$

It is clear that $B(\sigma) \in C^\infty(0, 1)$ and $B(\sigma)(1-\sigma)^{-1} \to 2^{1/3}$ as $\sigma \to 1-0$. After replacement of the variable of integration we obtain

$$\eta_0(x,t)=\frac{1}{4\pi}\int_{-\infty}^{\infty} e^{-it(\zeta^3/3-B(\sigma)\zeta)}\frac{F[\nu(\zeta)]\,d\zeta}{\zeta_\nu[\nu(\zeta)]}+O\left(t^{-N}\right)\ (t\to\infty),$$

$$\zeta_\nu^{-1}[\nu(\zeta)]=\frac{\zeta^2-B(\sigma)}{\sigma-\Omega_1'[\nu(\zeta)]}.$$

The function $\zeta_\nu^{-1}[\nu(\zeta)]$ is even. Consequently, near the points $\zeta = \pm B^{1/2}(\sigma)$ which yield the main contribution to the asymptotic $\eta_0(x, t)$ as $t \to \infty$, it can be represented in the form

$$\zeta_\nu^{-1}[\nu(\zeta)]=\sum_{j=0}^{\infty}a_j(\sigma)(\zeta^2-B(\sigma))^j.$$

Then

$$\eta_0(x,t)=\frac{1}{2\pi}a_0(\sigma)F(\nu_1(\sigma))\int_0^{\infty}\cos\left(\frac{1}{3}\zeta^3t-B(\sigma)\zeta t\right)d\zeta+\ \dots$$

$$(a_0(\sigma)=2^{1/2}B^{1/4}(\sigma)\,|\Omega''(\nu_1(\sigma))|^{-1/2}).$$

Finally, we find

$$\eta_0(x,t) = \frac{1}{2t^{1/3}} a_0\left(\frac{x}{t}\right) \mathrm{Ai}\left(-t^{2/3}B\left(\frac{x}{t}\right)\right) + \dots \tag{3.2}$$

as $t \to \infty$, $0 < x/t \le 1$ [Ai(z) is the Airy integral]. For $x/t = 1$ we have $B = 0$, $a_0(1) = 2^{1/3}$. In contrast to Eq. (1.3) obtained by the stationary phase method, the amplitude function $a_0(x/t)$ in Eq. (3.2) is bounded for $x/t \in [0, 1]$.

For $\sigma \ge 1$ we introduce a new variable k in place of ν, such that

$$(1/3)k^3 + D(\sigma)k = \nu\sigma - \Omega_1(\nu).$$

If $D(\sigma) > 0$ for $\sigma > 1$ and $D(\sigma) \to 0$ as $\sigma \to 1 + 0$, then the passage from ν over to k is nondegenerate. Replacement of the variable of integration yields

$$\eta_0(x,t) = \frac{1}{4\pi}\int_{-\infty}^{\infty} F[\nu(k)] \frac{k^2 + D(\sigma)}{\sigma - \Omega_1'[\nu(k)]} e^{-it(k^3/3 + D(\sigma)k)}\, dk + \dots$$

Here the phase function has a saddle point for $k = \pm iD^{1/2}(\sigma)$. Continuing the integrand into the complex plane $k \in C$ and expanding the amplitude function into a series in the neighborhood of the point $k = \pm iD^{1/2}(\sigma)$, we obtain

$$\eta_0(x,t) = \frac{1}{2t^{1/3}} a_0\left(\frac{x}{t}\right) \mathrm{Ai}\left(t^{2/3}D\left(\frac{x}{t}\right)\right) + \dots \tag{3.3}$$

as $t \to \infty$, $x/t \ge 1$. The function $D(\sigma)$ is selected such that the expression $k_\nu[\nu(k)]$ continued into the complex plane would be bounded and not equal to zero in the neighborhood of the poles $k = \pm iD^{1/2}(\sigma)$. We hence find

$$D(\sigma) = (3/2)^{2/3}(\sqrt{m \operatorname{tg} m} - \sigma m)^{2/3},$$

where $m = m(\sigma)$ is the least positive solution of the equation

$$(d/ds)\sqrt{s \operatorname{tg} s}\,|_{s=m(\sigma)} = \sigma.$$

We have

$$a_0(\sigma) = \lim_{k \to iD^{1/2}(\sigma)} \frac{k^2 + D(\sigma)}{\sigma - \Omega_1'[\nu(k)]},$$

which yields

$$a_0(\sigma) = 2^{1/2}D^{1/4}(\sigma)|((d^2/ds^2)\sqrt{s \operatorname{tg} s})|_{s=m(\sigma)}|^{-1/2}.$$

Formulas (3.2) and (3.3) determine the principle term of the uniformly suitable asymptotic $\eta_0(x, t)$ for large times. For $t \sim \varepsilon^{-1}$ it is necessary to take account of the rough nature of the bottom, as is realized exactly as in [1] for the Whitham method. A system of equations agreeing with Eq. (1.5), but with the distinction that

$$E_1(\lambda, \alpha) = E(\lambda, \alpha) + (1/6\theta)[qS(qh) - 2\Omega^2(\alpha)]$$

should be utilized instead of $E(\lambda, \alpha)$, is used here to construct the shape of the free boundary behind the bow wave front. The asymptotic $\eta(x, t)$ as $\varepsilon \to 0$ has the form

$$\eta(x,t) = C_0(\lambda, \alpha) \operatorname{ch}(qh) \mathrm{Ai}[-((3/2)\theta)^{2/3}] + \dots. \tag{3.4}$$

The initial conditions for the appropriate system of six first-order ordinary differential equations are conserved by the previous (1.6) with the exception of the following

$$\theta = \lambda_0[2\Omega^2(\alpha) - \alpha S(\alpha)],$$
$$C_0 = 3^{1/6}|\Omega(\alpha) - \alpha\Omega'(\alpha)|^{1/6} \operatorname{ch} \alpha F(\alpha)/[2(\lambda_0\Omega(\alpha))^{1/3}|\Omega''(\alpha)|^{1/2}],$$

which is due to the necessity of matching the solution with the asymptotic (3.2) for $\lambda = \lambda_0$ ($\lambda_0 \ll 1$). To construct the free surface shape ahead of the bow wave front it is necessary to solve an analogous system of equations [1] in which the hyperbolic functions are replaced by related trigonometric functions

$$\frac{dq}{d\lambda} = \frac{q^2h'}{\cos^2 qh},\ \frac{d\theta}{d\lambda} = 2\chi^2(\alpha) - qP(qh),\ \frac{dx}{d\lambda} = -P(qh),\ \frac{dC_0}{d\lambda} = -E_2(\lambda,\alpha)C_0,$$

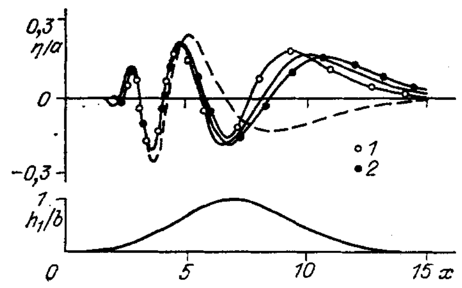

Fig. 3

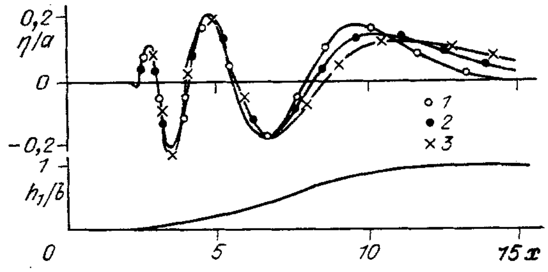

Fig. 4

$$\frac{dq_\alpha}{d\lambda} = \frac{q}{\cos^2 qh}\left[2q_\alpha h' + qh'' x_\alpha + 2qh' \operatorname{tg} qh\,(q_\alpha h + qh' x_\alpha)\right],$$

$$\frac{dx_\alpha}{d\lambda} = -P'(qh)(q_\alpha h + qh' x_\alpha),\ \chi(\alpha) = \sqrt{\alpha \operatorname{tg} \alpha},\ P(\xi) = \operatorname{tg}\xi + \frac{\xi}{\cos^2 \xi},$$

$$P_1 = \frac{P(qh)}{4\chi^2(\alpha)} - h \operatorname{tg} qh,\ E_2 = q_x [P_1 P(qh) - h] - qh' + \frac{q^2 h'}{\cos^2 qh} P_1 + \frac{1}{6\theta}[qP(qh) - 2\chi^2(\alpha)]. \quad (3.5)$$

Initial conditions that permit continuous matching of the solution of the system to the asymptotic (3.3) should be appended to this system

$$q = \alpha,\ \theta = \lambda_0 [2\chi^2(\alpha) - \alpha P(\alpha)],\ x = -\lambda_0 P(\alpha),$$

$$C_0 = \frac{3^{1/6} \,|\chi(\alpha) - \alpha\chi'(\alpha)|^{1/6}}{2[\lambda_0 \chi(\alpha)]^{1/3} \,|\chi''(\alpha)|^{1/2}} F_1(\alpha),$$

$$q_\alpha = 1,\ x_\alpha = -\lambda P'(\alpha) \quad \left(\lambda = \lambda_0,\ F_1(\alpha) = \int_{-\infty}^{\infty} f(x)\, e^{\alpha x}\, dx\right).$$

It is assumed that $f(x)e^{\pi|x|/2}$ is a function integrable in R^1. After solving the system (3.5) the function $\eta(x, t)$ ahead of the front is evaluated from the formula

$$\eta(x, t) = C_0(\lambda, \alpha) \cos qh \operatorname{Ai}[((3/2)\theta)^{2/3}] + \ldots \quad (3.6)$$

The principle terms of the asymptotics (3.4) and (3.6) agree on the bow wave front and equal

$$\eta(x, t) = \frac{F(0)}{2^{2/3} h^{1/4}(x)} M^{-1/3}(x) \operatorname{Ai}\left[\left(\frac{2}{M(x)}\right)^{1/3} (N(x) - t)\right] + \ldots, \quad (3.7)$$

where $M(x) = \int_0^x h^{1/2}(\xi)d\xi$, $N(x) = \int_0^x h^{-1/2}(\xi)d\xi$.

4. Comparing the different approximation with the exact solution for the case of a level bottom is represented in Fig. 2 ($x = 3$). The initial elevation of the free boundary is taken in the form (2.2) for $d = 3$. The solid curve shows the exact solution obtained as a result of numerical integration by Eq. (1.2), curves 1-3 are constructed from Eqs. (1.3), (3.2), and (3.3), respectively. It is seen that the asymptotics (3.2) and (3.3) describe the behavior of the free boundary well even for moderate times while Eq. (1.3) is applicable after the point of observation has passed the bow wave. The Whitham approximation (3.7) that corresponds to the long-wave approximation is denoted by the dashed line. A comparison with the exact solution shows that at distances from the initial perturbation that are comparable to the pond depth the long-wave approximation yields a qualitatively true pattern of the free boundary for the bow wave and is not applicable after the point of observation has passed it.

The influence of bottom relief on the free surface shape is shown in Figs. 3 and 4. The relationships (3.4) and (3.6) are used, and the bottom shape is indicated in the figures. For the localized bottom roughness described by the equation $h(x) = 1 - b \exp[-\beta(x - x_0)^2]$ for $\beta = 0.1$, $x_0 = 7$, $b = \pm 0.3$ (curves 1 and 2), the free surface shape at the time $t = 12$ is indicated in Fig. 3. The front boundary in these two cases corresponds to $x_f = 11.0$, 12.73. The exact solution for the level bottom is displayed by the solid line, and the solution for an infinite fluid by dashes. The influence of bottom roughness appears mainly in the neighborhood of the bow wave. A hillock ($b = 0.3$) on the bottom here results in retardation of this wave and a certain magnification of its amplitude. Passing over a trough ($b = -0.3$) the bow wave accelerates and its amplitude diminishes as compared with the case of a level bottom. It is interesting to note that the second hump of the inflowing wave is described sufficiently well by the solution of the problem for a level bottom even if it is directly above an obstacle but is not described by the solution of the corresponding problem for an infinite fluid. This indicates that the influence of bottom roughness on the free boundary shape appears not directly above an obstacle but is shifted in the direction of perturbation propagation.

The free surface shape for a bottom profile described by the function $h(x) = 1 - b \times [1 + \tanh\beta(x - x_0)]/2$ at the time $t = 12$ is represented in Fig. 4. Here $x_0 = 7$, $\beta = 0.33$, $b = \pm 0.3$; -1 (curve 1-3), and the front boundaries are $x_f = 11.21, 12.75, 14.46$. All the assertions referring to the localized roughness are valid even in the case of a smooth passage from one depth to another (see Fig. 4). The steepness of the wave will be smaller during emergence of the bow wave in the large depth domain, the greater the drop in depth.

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SOLUTION OF THE PROBLEM OF IDEAL FLUID FLOW IN THE NEIGHBORHOOD OF BODY AND WING APICES

A. V. Voevodin and G. G. Sudakov

UDC 532.5

For a uniformly accurate description of ideal fluid flow around three-dimensional bodies, the nature of its asymptotic behavior must be known in the neighborhood of the singular points that are the body and wing apices, for example. It is known that in the neighborhood of sharp apices the flow potential depends as a power-law on the distance to the apex.

An algorithm is proposed in this paper to solve eigenvalue problems by using the method of "vortex frames" and a panel method that permit finding the eigenvalues of the exponent and eigenfunctions of the problem. Examples are presented of application of the proposed method for problems of the flow around delta wing apices and apices of a body in the form of a circular cone that have an exact solution (the problems reduce to solving an ordinary differential equation). A comparison is given between the results of computations and the exact solutions.

1. Let us examine the problem of irrotational ideal fluid flow around a body apex or a wing angular point with half-angle θ at the apex. Let us introduce a Cartesian rectangular x, y, z coordinate system with x axis directed along the line of body (wing) symmetry, z axis in the plane of the wing (in the case of a cone, arbitrarily but perpendicular to the x axis), and y axis perpendicular to the x and z axes. The potential of the flow being investigated should satisfy the three-dimensional Laplace equation with boundary conditions of nonpenetration on the body (wing) surface. By virtue of the boundary conditions the problem is self-similar and, following [1-3], we seek its solution in the form

$$\Phi = Cx^n\varphi(y/x, z/x, \theta) \tag{1.1}$$

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